Seminar: Inverse Problems

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What is wave scattering?

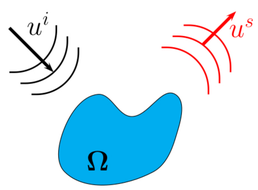
The task of wave scattering is to analyze the effect an object has on a wave when both collide. Scattering problems arise in various different applications, for example in computer tomography, radar or historically in Rutherfords discovery of atomic nuclei.

Mathematically, the problem can be described using partial differential equations. In the so called *direct scattering problem*, given an incident wave u^i, and an object Ω, we are interested in computing the scattered wave u^s. Provided the incident wave is a time *acoustic* plane wave $u^i(x,t) = \exp(i(kxd - \omega t))$ with wave number k, frequency ω and direction of propagation d, the total field $u = u^i + u^s$ can be described by the *Helmholtz equation*

$$-\Delta u - k^2 u = f \qquad x \in \mathbb{R}^d \backslash \overline{\Omega}. \tag{1}$$

Depending on the boundary condition one speaks about *sound soft* (Dirichlet b.c.) or *sound hard* (impedance b.c.) obstacle.

A second interesting case is given by using time-harmonic electromagnetic plane-waves as incident waves, which leads to studying the *Maxwell equations*.



Wave scattering.

Inverse Scattering Problem

In applications like radar, not the scattered field u^s is of interest, but rather the shape of the object Ω, which caused the scattering effect. This leads to so called *inverse problems*, where knowledge of the scattered field u^s is used to reconstruct Ω.

Solving the inverse scattering problem is considerably more difficult than solving the direct problem, as it involves nonlinear and ill-posed problems. An aim of this seminar is to present techniques, like regularization methods, to deal with these difficulties.

Prerequisites

This seminar is intended for bachelor, master and doctoral students. Minimum requirements however should be Functional Analysis 1, Differential Equations 1 and Numerical Analysis.

Preliminary Meeting

Monday, 11.3. 14:00, Freihaus seminar room DB gelb 04, 4th floor yellow area.

Possible Topics

Before studying the inverse problem, the direct problem has to be understood - analytically and numerically - first. Possible topics for talks can be about:

- Acoustic scattering: Helmholtz equation
- Direct obstacle scattering
- Numerical methods for the direct problem
- Ill posed problems
- Inverse scattering problems
- Waves in inhomogeneous media
- Electromagnetic scattering: Maxwell equations
- The inverse medium problem
- Numerical methods for the inverse problem

The above list can be extended or shortened taking the number of participants into account.

Examination modalities

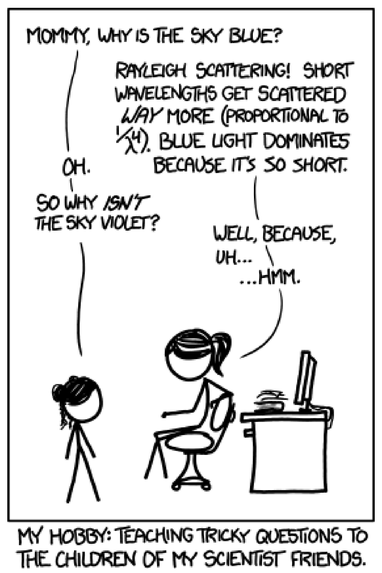
For a positive certificate a (approximately) 90-minute talk on the topic is mandatory. Furthermore bachelor students should prepare a written seminar paper. Other listeners are always welcome.

Some References

[CK13] David Colton and Rainer Kress. *Inverse acoustic and electromagnetic scattering theory*, volume 93 of *Applied Mathematical Sciences*. Springer, New York, third edition, 2013.

[Gol81] C. I. Goldstein. The finite element method with nonuniform mesh sizes applied to the exterior Helmholtz problem. *Numer. Math.*, 38, 1981.

[KM94] A. Kirsch and P. Monk. An analysis of the coupling of finite-element and Nyström methods in acoustic scattering. *IMA J. Numer. Anal.*, 14(4):523–544, 1994.



Rayleigh scattering
Image taken from: xkcd